

Relativity principle with a low energy invariant scale

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The possibility of a modification of special relativity with an invariant energy scale playing the role of a minimum energy is explored. Consistency with the equivalence of different inertial frames is obtained by an appropriate choice of a non-linear action of the Lorentz group on momentum space. Limits on the low energy cutoff from tests of Einstein's theory and possible ways to measure the new energy scale are discussed.

INTRODUCTION

In recent years a new cosmological paradigm has emerged, in which our Universe is filled with a “dark energy” that causes an accelerated expansion [1]. The nature of the dark energy is not known. However, a simple explanation is the existence of a cosmological constant, or, equivalently, a vacuum energy density, which, according to experimental data, would be of the order

$$\rho_V \sim (10^{-3} \text{ eV})^4. \quad (1)$$

The main advantage of this approach to dark energy is that the presence of a cosmological constant can be accommodated both in general relativity and in quantum field theory. However, in classical general relativity the cosmological constant is a completely free dimensionful parameter, while from the point of view of particle physics, it measures the energy density of the vacuum, that is, of the state of lowest energy, which makes it possible to give estimates for its value. If we are confident that we can use ordinary quantum field theory all the way up to the Planck scale, $M_P \sim 10^{19} \text{ GeV}$, then we expect that

$$\rho_V^{(\text{Planck})} \sim (10^{28} \text{ eV})^4. \quad (2)$$

There is then a mismatch of 124 orders of magnitude between the expected and the measured values of the vacuum energy density, or of 31 orders of magnitude between the Planck mass and the energy scale which defines the experimental value of ρ_V . Even if one thinks that the energy cutoff of quantum field theory should be changed from the Planck scale to a lower mass scale, the discrepancy would still be of many orders of magnitude. In addition, one expects contributions to ρ_V coming from the different electroweak, chiral or even GUT phase transitions, of the order of the corresponding energy scales at which these transitions took place in the early universe. It is very difficult to think why all these apparently unrelated contributions should almost cancel to produce the observed value Eq. (1).

An alternative to trying to derive the mass scale which controls the vacuum energy density in terms of other measurable parameters is to postulate that it is a fundamental energy scale, just like c is a fundamental velocity scale.

Also there is a general agreement that in a quantum theory of gravity the maximum entropy of any system should be proportional to the area and not to the volume [2, 3] which implies the breakdown of any effective field theory to describe systems which exceed a certain critical volume. This observation leads [4, 5] to endow the effective field theory with an *ojo* infrared (IR) cut-off. Recently a possible connection between the matter-antimatter asymmetry of the universe and a violation of CPT parametrized by a low energy scale has been pointed out [6].

All these arguments lead to consider a new low energy scale as a possible relic in the microscopic world of the gravitational interactions.

However, an energy scale is not invariant under Lorentz transformations between inertial observers. One could ask whether it would be possible to find a modified relativity principle in which all inertial observers could agree on the value of the cosmological constant as a fundamental quantity.

In fact attempts along the line to generalize special relativity to include the invariance of an energy scale have been started to be explored recently in the framework of the so-called “Doubly special relativity” [7, 8]. Motivated by quantum gravity ideas, the invariant energy scale has been assumed to be the Planck mass in these attempts. The fact that the Planck mass is much larger than the energy scales explored so far in particle physics makes that the new kinematics is only a very small, high-energy correction to standard relativistic kinematics.

In this letter we are questioning however the possibility to find an “infrared doubly special relativity”, in the sense that we explore whether the presence of an energy scale of the order of 10^{-3} eV , which in principle could be thought to crudely affect even to nonrelativistic kinematics, could be compatible with the standard physics we know.

In the attempts to incorporate the Planck energy as a new invariant in a relativistic theory the main ingredient is to consider a non-linear realization of Lorentz transformations in momentum space. This suggests to use the same idea in order to incorporate a new invariant low energy scale. The simplest way to do that is to take the non-linear action of Lorentz transformations in DSR2 [8] which leads to an invariant scale as an ultraviolet cutoff

for the energy and replace everywhere the energy by its inverse. In this way one gets automatically a non-linear action of Lorentz transformations in momentum space with an invariant energy scale which now corresponds to an infrared cutoff as we were looking for. Although this simplest first choice will turn out to be phenomenologically inconsistent it is convenient to discuss first this case because most of the results can be applied to a simple modification which avoids these inconsistencies.

The explicit form of the non-linear Lorentz transformations is defined by the relations

$$p_0 = \pi_0 + \lambda \quad \mathbf{p} = \boldsymbol{\pi} \left(1 + \frac{\lambda}{\pi_0} \right) \quad (3)$$

where $\pi_0, \boldsymbol{\pi}$ is a linearly transforming auxiliary four-momentum and p_0, \mathbf{p} the physical four-momentum. From these relations one gets

$$\pi_0 = p_0 - \lambda \quad \boldsymbol{\pi} = \mathbf{p} \left(1 - \frac{\lambda}{p_0} \right) \quad (4)$$

which makes manifest the role of the invariant energy scale λ as a minimum for the physical energy ($p_0 > \lambda$). The standard relativistic invariant mass-shell condition for the auxiliary four-momentum ($\mu^2 = \pi_0^2 - \boldsymbol{\pi}^2$) can be reexpressed in terms of the physical four-momentum as

$$\mu^2 = \left(1 - \frac{\lambda}{p_0} \right)^2 (p_0^2 - \mathbf{p}^2) . \quad (5)$$

BOUNDS FROM QED TESTS

Our observations of electromagnetic radiation going down to frequencies of the order of Hz or energies of the order of tens of feV exclude the possibility to have a non-linear representation of Lorentz transformations like (3) for photons unless the scale λ takes an unobservable small value.

Then in order to proceed one has to consider a generalized relativity principle which does not affect the kinematics of massless particles. There is a very simple way to modify (3) in such a way that only massive particles are sensitive to the low energy scale. One can consider a nonlinear realization of Lorentz transformations based on

$$p_0 = \pi_0 \left[1 + \frac{\lambda}{\pi_0} f \left(\frac{\pi_0^2 - \boldsymbol{\pi}^2}{\lambda^2} \right) \right] \quad (6)$$

$$\mathbf{p} = \boldsymbol{\pi} \left[1 + \frac{\lambda}{\pi_0} f \left(\frac{\pi_0^2 - \boldsymbol{\pi}^2}{\lambda^2} \right) \right] \quad (7)$$

with a function f such that $f(0) = 0$ and $f(\infty) = 1$. The first condition on f leads to linear Lorentz transformations for massless particles and the second one justifies

the approximation in (3) for particles whose mass is much larger than the low energy invariant scale. This will be the case for all the known elementary particles with the only possible exception of neutrinos for which it may be necessary to consider the exact expression (6-7). In fact the net effect of the modification of the relativity principle is to replace the universal scale λ , in the kinematics of any particle with a mass parameter μ , by

$$\lambda(\mu) = \lambda f \left(\frac{\mu^2}{\lambda^2} \right) \quad (8)$$

which approaches λ when $\mu \gg \lambda$.

For all the particles where there is a good control of the approach to the non-relativistic limit (i.e. for all the particles except neutrinos) it is clear that the mass scale μ should be much greater than the new scale λ in order to reproduce the very well tested relativistic corrections to the non-relativistic limit. In fact since the corrections due to the new scale are proportional to the ratio λ/p_0 the best place to look for a signal of the new scale is through a small deviation from the relativistic corrections as predicted by special relativity. An expansion in powers of the new scale λ leads to a modification of the dispersion relation which to first order in λ is given by

$$p_0 \simeq \sqrt{\mathbf{p}^2 + \mu^2} + \frac{\lambda(\mu)}{1 + \frac{\mathbf{p}^2}{\mu^2}} . \quad (9)$$

In the non-relativistic limit one has

$$p_0 \simeq m + \frac{\mathbf{p}^2}{2m} - \frac{\mu}{m} \frac{\mathbf{p}^4}{8m^3} \quad (10)$$

where we have introduced a physical mass parameter

$$m = \mu + \lambda(\mu) . \quad (11)$$

We see that the first two terms are the same as in the non-relativistic limit of special relativity, the effect of the new scale λ being reduced to a redefinition of the physical mass m as a combination of the infrared scale λ and the auxiliary mass parameter μ . But we have a correction in the coefficient of the next order term by a factor μ/m and by applying this kinematical analysis to the electron we would find a slight modification of the energy levels of the hydrogen atom. Given the extraordinary agreement between theory and the experimental measurement of the Lamb shift (one part in 10^5 [9, 10]) we get a bound on the low energy scale $\lambda \lesssim 5$ eV.

The most stringent bound on the low energy scale that can be obtained from QED tests comes from the possible modification induced by the presence of such scale in the determination of the anomalous magnetic moment of the electron. If one uses the Dirac equation in terms of the auxiliary variables one can show that at tree level all the effect of the low energy scale can be reabsorbed in the mass parameter m and then no anomalous magnetic moment (a_e) is generated in this approximation. A

simple estimate of the correction to the usual calculation imposed by the low energy scale is

$$\delta a_e \sim \frac{\alpha}{\pi} \left(\frac{\lambda}{m_e} \right) \sim 4 \times 10^{-9} \frac{\lambda}{(1 \text{ eV})}. \quad (12)$$

If we ask this correction to be smaller than the uncertainty of the theoretical prediction for a_e in QED caused by the uncertainty in the determination of α [9], we get a bound for the IR scale $\lambda \lesssim 10^{-2} \text{ eV}$.

BOUNDS FROM NEUTRINO PHYSICS

Another way to look for bounds on the low energy scale λ is to consider neutrinos. In this case masses can be of the order of the low energy scale and kinematical corrections can be more important.

One indirect bound on the low energy scale comes from the contribution of neutrinos to the energy density of the Universe [11] which puts a bound on the physical mass parameter m of the neutrino. If one assumes three degenerate Dirac neutrinos one gets $m < 4 \text{ eV}$. Since the low energy scale $\lambda(\mu)$ is bounded by m (as a consequence of the definition of the physical mass parameter in (11) and the condition $\mu > 0$) then the upper bound on m is directly an upper bound on $\lambda(\mu)$.

Another indirect bound on the low energy scale comes from experiments trying to measure or put limits on neutrino masses from the tritium beta-decay spectrum which at present [11] are at the level of the eV. The bound on the low energy scale will be at best of the same order of magnitude.

Then the indirect bounds on the low energy scale from bounds on neutrino masses are less stringent than the bound obtained from QED tests.

If one tries to get direct bounds from the observations of neutrinos there is an additional difficulty to identify a signal of the low energy scale. Only neutrinos whose energy is much larger than its mass can be detected and once more the kinematical corrections due to the low energy scale are suppressed. We need in this case the ultra-relativistic limit of (9):

$$p_0 \simeq |\mathbf{p}| + \frac{\mu^2}{2|\mathbf{p}|} + \frac{\lambda(\mu)\mu^2}{p^2}. \quad (13)$$

The phase of the oscillating amplitude between two neutrino states with mass parameters μ_1 and μ_2 will be

$$\theta_{12} = L(E_1 - E_2) \approx L \frac{\mu_1^2 - \mu_2^2}{2|\mathbf{p}|} + L \frac{\mu_1^2 \lambda(\mu_1) - \mu_2^2 \lambda(\mu_2)}{p^2}. \quad (14)$$

If one has an experiment with the ratio $|\mathbf{p}|/L$ of the order of $\mu_1^2 - \mu_2^2$ one will observe the oscillation between the two types of neutrinos as expected in special relativity and the correction due to the infrared scale λ will be

unobservable. In order to have a signal of the infrared scale in neutrino oscillations one requires an experiment with \mathbf{p}^2/L of the order of $\mu_1^2 \lambda(\mu_1) - \mu_2^2 \lambda(\mu_2)$. The ratio \mathbf{p}^2/L can be written as

$$\frac{\mathbf{p}^2}{L} \approx 2 \times 10^{-6} \text{ eV}^3 \left(\frac{|\mathbf{p}|}{\text{MeV}} \right)^2 \left(\frac{10^8 \text{ Km}}{L} \right). \quad (15)$$

Then one sees that solar neutrino experiments are the appropriate place to look for a signal of the low energy scale or for a relevant bound on it.

SUMMARY AND DISCUSSION

If one wants to go beyond this simple kinematical analysis of possible signatures of a low energy cutoff one should have to consider a dynamical theory where the non-linear Lorentz transformations, as defined by (3), were realized on the space of states. This is an open problem which has been the subject of recent work [12, 13] in connection with a modified relativity principle with a high energy invariant. It is not possible at present to do a dynamical analysis, it is not even clear whether there can be an obstruction to construct a dynamical theory with a modified relativity principle compatible with an invariant energy scale.

Another limitation of the present work is that all the analysis has been concentrated on the non-linear realization of Lorentz transformations based on the introduction of the physical momentum variables as in (3). This is not the only possible way to introduce a low energy scale. It corresponds to a translation to low energies of a similar construction at high energies [8] but there are many other possibilities. We expect that the main conclusions are not a peculiarity of the choice of the non-linear transformations used and that they will apply to a more general case.

To summarize we have found that it is possible to introduce a low energy invariant scale in a relativistic theory and that if this new scale is of the order of 10^{-3} eV it can be made compatible with all the tests of special relativity and at the same time one is close to detect its signals in different experiments. If one assumes a mechanism of cancellation for the contributions to the vacuum expectation value of the energy momentum tensor then on dimensional grounds one would expect

$$\langle T_{\mu\nu} \rangle \sim \lambda^4 \left(c_1 \delta_\mu^0 \delta_\nu^0 + c_2 \sum_i \delta_\mu^i \delta_\nu^i \right) \quad (16)$$

with c_1, c_2 dimensionless coefficients depending on the details of the theory incorporating the new low energy scale. Then the present acceleration of the expansion of the Universe could be a signal of a new low energy scale compatible with a modified relativity principle.

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